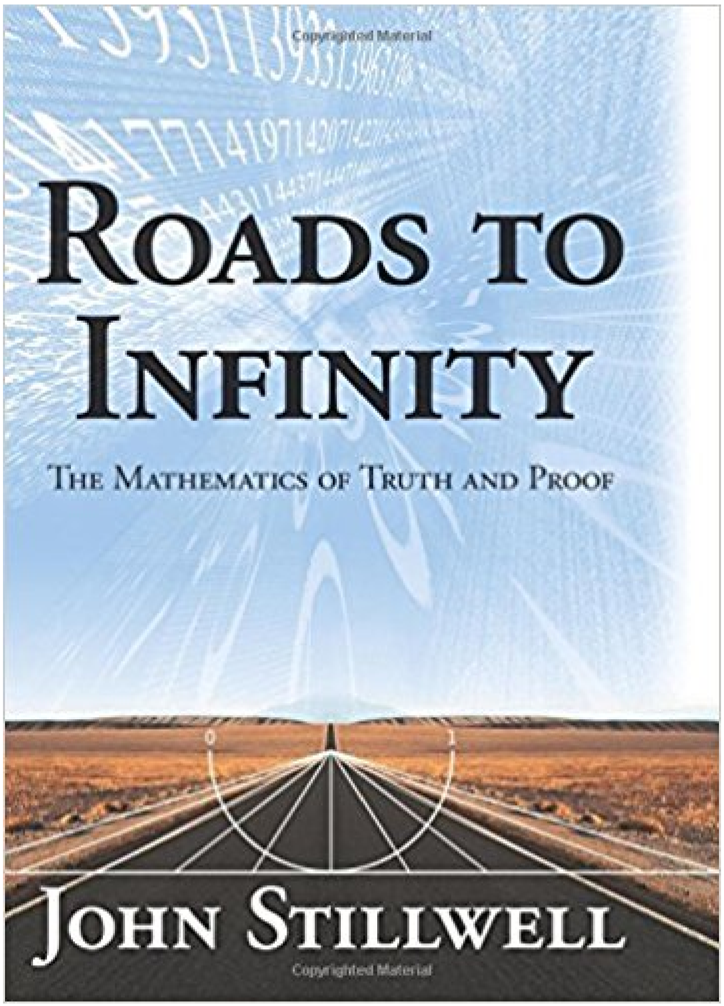
Roads To Infinity: The Mathematics Of Truth And Proof



Synopsis

Winner of a CHOICE Outstanding Academic Title Award for 2011!This book offers an introduction to modern ideas about infinity and their implications for mathematics. It unifies ideas from set theory and mathematical logic, and traces their effects on mainstream mathematical topics of today, such as number theory and combinatorics. The treatment is historical and partly informal, but with due attention to the subtleties of the subject. Ideas are shown to evolve from natural mathematical questions about the nature of infinity and the nature of proof, set against a background of broader questions and developments in mathematics. A particular aim of the book is to acknowledge some important but neglected figures in the history of infinity, such as Post and Gentzen, alongside the recognized giants Cantor and GÃ¶del.

Book Information

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Customer Reviews

"The text is nicely presented, with many illuminating and/or entertaining quotes and diagrams. â | The wide scope of the work and the way that the author manages to show the connections between the different topics are the main strength of the book, making it a good place to start if you have some interest in logic, set theory, or just challenging ideas in general."â •Yann Peresse, London Mathematical Society Newsletter, June 2013 "The book follows essentially two roads to infinity: Cantorâ ™s diagonal argument and Cantorâ ™s construction of the ordinals. Stillwell shows how these two themes intertwine and influence a wide range of mathematical questions â | The scope of this book is breathtaking, but Stillwell has masterfully presented and developed a wide range of mathematics as a coherent narrative. â | He is able to pack a lot of information and ideas into a few well-chosen paragraphs without sacrificing clarity. Stillwell is an accomplished historian of mathematics who doesnâ ™t limit himself to the work of the well-known. I appreciated the appearance of some of the lesser known contributors to the study of the infinite. â | It is well-conceived and well-written, and covers a large amount of material on logic, transfinite set theory, provability, combinatorics, and the histories of these fields."â •James V. Rauff, Mathematics and Computer Education, Winter 2012 "Stillwell is a master expositor and does a very good job explaining and weaving together many core issues in mathematical logic and foundational studies. â | Stillwellâ ™s book is highly commendable, very informative and well organized. It is very carefully produced."â •JosÃ© FerreirÃ s, American Mathematical Monthly, February 2012 "I highly recommend it for undergraduates in mathematics and other young mathematicians who are looking for historical context or a different angle to their studies. Readers who have experience with theoretical analysis or a foundation in abstract mathematics will find the examples wonderfully illustrative. For these readers, Stillwellâ ™s words will flow smoothly, almost like a novel."â •Joyance Meechai, Mathematics Teacher, October 2011 "Stillwell has produced an excellent book on infinity for the motivated lay reader. â | The author does a masterful job of painting a historical portrait of logic, set theory, incompleteness, computable functions, and many associated foundational questions. His lively style and clear exposition of the relationship between proof and truth will engage both the novice and the expert. Although there are numerous books on the topic of infinity, Stillwell tells a story which motivates the ideas he introduces. This is a book that anyone with an interest in mathematics should have in their library. Highly recommended."â •R.L. Pour, CHOICE, March 2011 "This book is an accessible, but also a scholarly and extremely well-written introduction to the great ideas of modern logic. While the central results are the famed proofs of GÃ¶del, Stillwell does a masterful job of relating that work not only to GÃ¶delâ ™s contemporaries, such as Post, Turing, Church, Tarski, Gentzen, and von Neumann, but also to modern researchers in the foundations of mathematics (Friedman, Woodin, and others). Chapter 6 on natural unprovable sentences is a gem â | Stillwellâ ™s book is worthwhile reading for anyone interested in the development of mathematical logic in the 20th century and learning about the possible directions of the field in the 21st."â •Stan Wagon, The College Mathematics Journal, March 2011 "In 1963, Edwin E. Moise published Elementary Geometry from an Advanced Standpoint and his book became a classic. â | [this book] deserves the same outcome. â | One of the most enjoyable features is Stillwellâ ™s use of techniques of logic and set theory to solve real mathematical problems â | Another enjoyable feature is Stillwellâ ™s uniform coverage of unprovability, undecidability and non-computability â | suitable for self-study â | it is excellent background material for computer scientists and mathematicians in other fields. The historical notes alone are worth perusing by anyone who is interested in the development of mathematical ideas."â •Phill Schultz, Gazette of the Australian Mathematical Society, March 2011 "â | a clear and succinct guide. â | One interesting feature of the book is the careful treatment of two of the less famous contributors in this areaâ •Emil Post and Gerhard Gentzen â |"â •CMS Notes, Vol. 43, No. 1, February 2011 "â | excellent book â | the investment the reader makesâ •be he an intellectually curious adult or a math grad student with extra time on her handsâ •pays off with an increased understanding of the fascinating world of mathematical logic. The authorâ ™s thorough, well-researched historical comments are particularly valuable, as well as the philosophical quotations from the important players in this game. There is a very complete bibliography. What the reader might appreciate most is the ability of the author to share his deep insights into what is important and what it all means in the most profound sense. â | it is clear that the book received excellent proofreading before publication. â |"â •Mathematical Reviews, Issue 2011f "This is an interesting book on infinity. The author combines set theory and logic to face the most basic and fruitful aspects of infinity."â •Claudi Alsina, Zentralblatt MATH 1196 "Featuring chapters dedicated to the diagonal argument, ordinals, computability and proof, logic, arithmetic, natural unprovable sentences, and axioms, as well as being enhanced with the inclusion of a lengthy bibliography and a comprehensive index, Roads to Infinity: The Mathematics of Truth and Proof is highly recommended reading for students, scholars, and non-specialist general readers with an interest in the history and contemporary issues of mathematics today."â •Able Greenspan, Midwest Book Review "I love reading anything by John Stillwell. If you've ever been tantalized by the puzzles of infinity, set theory, and logic, and want to understand what's really going on, this is the book for you. It's an exceptionally fine piece of mathematical exposition."â •Steven Strogatz, Cornell University, author of The Calculus of FriendshipÂ

John Stillwell was born in Melbourne, Australia in 1942 and educated at Melbourne High School, the University of Melbourne (M.Sc. 1965), and MIT (Ph.D. 1970). From 1970 to 2001 he taught at Monash University in Melbourne, and since 2002 he has been Professor of Mathematics at the University of San Francisco. He has been an invited speaker at several international conferences, including the International Congress of Mathematicians in Zurich 1994. His works cover a wide spectrum of mathematics, from translations of classics by Dirichlet, Dedekind, Poincare, and Dehn to books on algebra, geometry, topology, number theory, and their history. For his expository writing he was awarded the Chauvenet Prize of the Mathematical Association of America in 2005, and the AJCU National Book Award in 2009. Recent titles by Stillwell include Yearning for the Impossible, Mathematics and Its History, The Four Pillars of Geometry, and Geometry of Surfaces.

John Stillwell's "Roads to Infinity" sounds like another "GÃ¶del's theorem" book. Yes, it mentions it and gets into GÃ¶del's famous theorems, a little bit. But, he just finds a simple alternative; he doesn't mention GÃ¶del numbering. I've been through say Nagel's "GÃ¶del's Proof." But, even then, I don't recall all this stuff about ordinals. John Stillwell shows George Cantor's genius a lot more by showing the Fourier analyses roots of his ideas and George Cantor's ordinal numbers work. This is just a start of what you can get from here, if you're not a phd logician/mathematician.If you think George Cantor's transfinite numbers is mindblowing, his work on ordinals to analyse infinit cardinal numbers is . . . it shows how one can analyse infinity far more than most people could ever dream.I'd like to say, that as usual, in John Stillwell's works, he always finds some modern easier short proof of results; otherwise, he references off. He gives examples when he can find easy examples, and just mentions hard problems(mostly in chapter supplements where they belong).In John Stillwell's other works, he loves to point out the induction definitions of arithmetic; in this book, he gives a lot more reason to take those seriously!Historically, one of the major reasons for proof was the inability to deal with infinity. Here, John Stillwell shows the affect infinity has had on proof. I think logical proof has proven to be more than we thought through the ages, so I don't think one should consider syllogisms discredited by modern studies of infinity and proof. But, I do like the recent results of the interplay between finite and infinit. And John's book . . . well, I don't think he does more than introduce one to these things.In Oyestein Ore's "Number theory and its History", he gets into some number theory, some diophanine analyses, where he shows some lead to infinity and some are finite. This has always suggested to me that determining when things are finite and when they are infinity, when they are compact or not seems to be an interesting question in mathematics(I can't say that I totally know lie theory; but, I have done some look ahead and found that some of the majory lie theory work of the twentieth century anyways has to do with compact and non-compact geometry). And so, the mathematics of the interplay between the finit and infinit at least excited me.

Whoa... I mean this is the kind of book that is so significant and intellect changing that you simply MUST read it, but.......and this is a big but, read slowly, take your time, and absorb. Utterly mind stretching..........I LOVED IT! Will be reading this many times......

This is an excellent introduction to how logic and modern set theory affect modern mathematics.

Delves into Cantor's theory of transfinite numbers. Also examines work of Kurt Godel, Emil Post , and Gerhard Gentzen. Not too technical. Very readable.

I bought this book out of interest since it had very good reviews from other readers. I was not disappointing as the book is very well written, informative, and interesting. The introductory chapters on the concept of infinity provide very clear explanations of the problems associated with defining what is the limit of very large numbers. This book I would highly recommend to anybody who wanted to read a non-technical account of the issues around infinity and to come away with a sound understanding of the what the term means.

It's again a bipolar mathematician? but not Cantor...Give a pension to him, make like....

Clear and very concise. Stillwell did a masterful job on this review of the mathematics of different infinities.The book is both content rich and also easy to understand.

The book's writing sometimes seemed vague and confusing. That said, it did have interesting content.

[The other "MidWest Book Review" seems to miss the main point of the book and doesn't do justice to it.]This book is not original research, but is still a great book because it opened my eyes to some very important maths about logic that I've overlooked. As the title says, it's about truth and proof. It surprised me that the "strength" of proof systems is somehow related to transfinite numbers.Chapter 1: Aleph_0 is the cardinality of the integers. 2^Aleph_0 is the cardinality of the continuum (ie, the real line). By Cantor's diagonal argument we know there is no 1-1 correspondence between the integers and the reals.Chapter 2: Cantor's theory of infinite ordinals. We can count from 1,2,3,... to infinity, and BEYOND that, is the first transfinite ordinal, that Cantor denotes as omega. Then we can carry on counting with omega + 1, omega + 2, omega +3, ..., to omega * 2. This process goes on to omega * 3, omega * 4, ..., and to omega^2, omega^3, ..., omega^omega, omega^omega^omega, ..., and eventually to omega raised to omega an infinite number of times, but it still doesn't end. The next ordinal is epsilon_0, and these countable ordinals go "inconceivably far beyond" epsilon_0. This results in Aleph_1, the first UNCOUNTABLE ordinal, and it still doesn't end!The continuum hypothesis asks whether 2^Aleph_0 = Aleph_1. It is still unsolved, but Cohen believes that it is highly unlikely to be true. Godel proved that CH is consistent with standard

Zermelo-Frankel set theory. Cohen (the inventor of "forcing") proved that it cannot be proved in ZF.All this is explained very clearly in the book; my summary is lousy. John Stillwell's writing style is very engaging, he knows the subject thoroughly and is able to explain every detail with exceptional clarity.Chapter 3: About Emil Post's efforts to search for a formulation of all formal systems. He saw that unprovability is a simple consequence of the diagonal argument; this predated Godel's incompleteness theorems, but he didn't publish because the Church-Turing thesis was not yet established at that time (so he wasn't sure if his normal form is universal; Later it turned out to be, of course, Turing-equivalent).Chapter 4: An introduction to logic and deduction, via Gentzen's sequent calculus. I mainly skimmed this chapter. Cut elimination is introduced here.Chapter 5: This chapter is very crucial. It starts with the Peano axioms for arithmetic (PA). We can assign a countable ordinal to each vertex of the proof tree. Thus, a proof system's "strength" can be measured by what kind of induction it allows. Gentzen 1943 proved that induction up to any ordinal less than epsilon_0 can be proved in PA. However, there exists "real" theorems whose proof lies beyond epsilon_0 induction. Some examples are given next...Chapter 6: "Natural Unprovable Theorems". Eg: the Paris-Harrington theorem in Ramsey theory and the Tao-Green theorem in number theory.Chapter 7: About "Axioms of Infinity" that can be added to ZF so it can deal with infinities. [I haven't read this chapter yet, maybe later. Hope this review helps you so far!]

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